**Papers written by**

**Australian Maths**

**Software**

**SEMESTER ONE**

**MATHEMATICS SPECIALIST REVISION 2**

**UNIT 3**

**2016**

**SOLUTIONS**

Mathematics Specialist Unit 3, 2016, Semester One Solutions

**Section One**

1. (6 marks)

2 2

1 1 1

*x x*

− +

11 1 1

+ ~~= =~~

(a) (1) 2 2 2

*x x x*

− − −

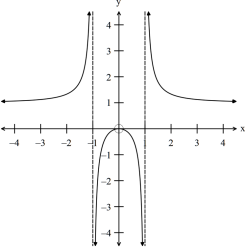
✔

(b) (5) ✔

✔

2. (10 marks) (a)

✔

✔ 

✔

1 2 3 15

⎡ ⎤

⎢ ⎥ − − −

1 1 1 3

2 1 1 9

⎣ ⎦

1 2 3 15

⎡ ⎤

⎢ ⎥ − 0 3 4 18

*R R*

✔

1 2

⎣ ⎦ − 0 3 5 21 2

*R R*

✔

1 3

1 2 3 15

⎡ ⎤

⎢ ⎥

0 3 4 18

⎣ − − ⎦ − 0 0 1 3

*R R*

✔

2 3

− = − → = *z z*

3 3

( ) ( )

3 4 3 18 2

*y y*

+ = →=

( ) ( )

*x x*

+ + = →=

2 2 3 3 15 2

( )

The point of intersection is 2 2 3 *, ,*

✔

(4)

2

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1 1 1 4

⎡ ⎤

⎢ ⎥ −

0 2 1 2

(b)

− − ⎣ ⎦

0 0 2 *p* 7 2*q* 12

(i) Exactly one solution if 2 *p* − 7 ≠ 0 ⇒*p* ≠ 3***.***5 ✔✔ (2) (ii) There is no solution if *p* = 3***.***5 and 2*q* −12 ≠ 0 i.e. *q*≠6✔✔ (2) (iii) There are inifinitely many solutions if *p* = 3***.***5 and *q*=6✔✔ (2)

3. (13 marks)

✔

(a) (*z* − (1+ 2*i*))(*z* − (1− 2*i*))(*z* − (3+ *i*))(*z* − (3− *i*)) = ⎡ − + − − ⎤⎡ − + − − ⎤ ( ( ))( ( )) ( ( ))( ( )) *z i z i z i z i*

1 2 1 2 3 3 ⎣ ⎦⎣ ⎦ 2 1 2*i* 1 2*i z z* 3 *i* ⎡ + + − ⎤ − +⎣ ⎦ ( +3−*i* )()()

⎡ ++−⎤⎣ ⎦

= − ( + *i* +1− 2*i* ) ( )( )

2

*z z*

1 2

33

*i i*

( )( )

✔

2 2 2 2

= − + − − + − *z z i z z i*

2 1 4 6 9

( )( )

2 2

= − + − +

*z z z z*

2 5 6 10

4 3 2

= − + − +

*z z z z*

8 27 50 50

✔

Therefore equation is 4 3 2

*z* −8*z* + 27*z* − 50*z* + 50 = 0

(3)

( ) ( )

3 2 3 5

b *Let P z z z z*

= − + +

( )

*P*

− = − − − + =

1 1 1 3 5 0

∴ = −

*z*

1

Using synthetic division with You can use long division but slower z = −1 ∴= −+

*z z*

1 1

3 2

*z z z*

soisafactor

− + +

3 5

2

*z z*

−+

− − 1 1 1 3 5

25

✔ method

)

3 2

*z z z* −++

35

*z*

+ 1

↓ − −

1 2 5

( )

3 2

− +

*z z*

1 2 5 ~~0~~

− 2

2

−+

2 3*z z*

∴ = − − + = *z z z*

1 2 5 0 ✔

OR

( )

2

− −−

2 2

2 4 20

± − *z*

= 2

2

*z z* 55

*z*

+

( )−+

55

*z*

2 16 2 16 ± − ±

*i*

*z*

= ~~=~~

✔

2 2

0

2 4

±

*i*

*z*

= 2 *z i or z*

2

*z z z* = − −+=1 250or

= ± = − 1 2 1 ✔

(c) (i) 4

*z* = −16

3

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( )

4

*z cis n n R* = π + × π ∈ 16 2

1

( ( ))

*z cis n*

= π + π

2 2

4

⎛ π π ⎞ = ~~+~~ ⎜ ⎟

*n*

*z ~~cis~~*

24 2

✔

⎝ ⎠

⎛ π ⎞ = ~~=~~ = + ⎜ ⎟ *n ~~z cis~~ i*

0 ~~2~~ 2 2

***,***

4

⎝ ⎠

⎛ π ⎞ = ~~=~~ = − + ⎜ ⎟

3

*n ~~z cis~~ i*

1 ~~2~~ 2 2

***,***

4

⎝ ⎠

⎛ π ⎞ = ~~=~~ ⎜ ⎟

5

*n ~~z cis~~*

2 ~~2~~

***,***

4

⎝ ⎠

⎛ π ⎞ = ~~− = −~~ = − ⎜ ⎟ *n ~~z cis~~ i*

1 ~~2~~ 2 2

***,***

4

⎝ ⎠

⎛ π ⎞ = ~~− = −~~ = − − ⎜ ⎟

3

*n ~~z cis~~ i*

2 ~~2~~ 2 2

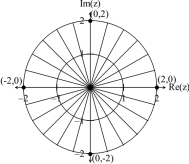
***,***

4

⎝ ⎠

✔✔ (3)

(ii)



✔✔ (2) ⎛ π π ⎞ = ~~+~~ ⎜ ⎟

(iii) is equivalent to 4

*n*

*z* = ~~−~~16 24 2

*z ~~cis~~*

⎝ ⎠

⎛ π⎞ = ~~+~~ ⎜ ⎟

*n*

whereas is equivalent to which means the 4 *z* =16 ~~2 0~~

*z ~~cis~~*

2

⎝ ⎠

starting positions of the roots are different apart.. 4π The roots themselves are apart and the roots of the two equations 2π start apart. ✔ (1) 4π

4

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4. (13 marks)

5 5 5

⎛ ⎛ π ⎞⎞ ~~⎛~~ ⎛ π ⎞⎞ ~~⎛~~ ⎛ π ⎞⎞ 5

( ~~)~~ ( ~~)~~

a *cis ~~i cis cis~~* ⎜ ⎜ ~~⎟~~⎟ + ~~− =~~ ⎜ ⎜ ~~⎟~~⎟ + ⎜ ⎜− ⎟⎟

1 ~~2~~

✔

4 4 4

⎝ ⎝ ⎠⎠ ⎝ ⎝ ⎠⎠ ⎝ ⎝ ⎠⎠ ⎛ π ⎞ ⎛ π ⎞ = ~~+ −~~ ⎜ ~~⎟ ⎜~~ ⎟

5 5

5

( ~~)~~

*cis ~~cis~~*

2

4 4

⎝ ⎠ ⎝ ⎠

1 1

*i i*

⎛ ⎞ = ~~− − + − +~~ ⎜ ⎟

4 ~~2~~

✔

2 2 2 2

⎝ ⎠

14 1

*ii*

( )

= ~~− −~~ + − +

✔

2 2

5

⎛ ⎛ π ⎞⎞ ⎛ ⎞ ⎛ ⎞

1 1

5

( ~~)~~

⎜ ~~⎟~~ + ~~− = − − + −~~ ⎜ ~~⎟~~ ⎜ ~~⎟ ⎜~~ ⎟ *cis ~~i~~ i*

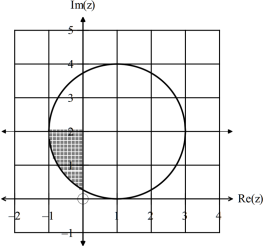
1 ~~4 4~~

4 2 2

⎝ ⎝ ⎠⎠ ⎝ ⎠ ⎝ ⎠

(3)

(b)

✔

✔

✔

(c) *z* +1 = *z* − *i* ✔✔ (2)

⎛ ⎛ π ⎞ ⎛ π ⎞⎞

⎜ ⎜ ~~⎟~~ + ⎜ ⎟⎟

*cos ~~isin~~*

(d) ( ) ✔ 3 3 4 ⎝ ⎝ ⎠ ⎝ ⎠⎠ ⎛ π π ⎞ = ~~=~~ ⎜ − ⎟ = −π

*z cis cis* 4 4 3 3

⎛ ⎛ π ⎞ ⎛ π ⎞⎞ ⎝ ⎠

⎜ ⎜ ~~⎟~~ + ⎜ ⎟⎟

*cos ~~isin~~*

3 3

⎝ ⎝ ⎠ ⎝ ⎠⎠

mod(*z*) =1 arg(*z*) = π ✔

5

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(e) (2) ( )

( )

*zi i* − − −

3 2 4 3 6 17

*i i i*

( ~~)~~ 625

= ~~× =~~

*Re ~~z~~* =

( )

( )

4 3 4 3 25

+ −

✔ ✔ ✔

5. (8 marks)

2 2 1

(a) ( ( )) ( ) ( ) − = − = − *g x* 1 *x f x x* 1 ✔

21

( ( )) ( )

− =

*g x f x*

2

( )

1 1

− = −

*x x*

2

( )

*i.e. x x*

− = −

1 1

✔

2

( ) ( )

*x x*

− − − =

1 1 0

( )[ ]

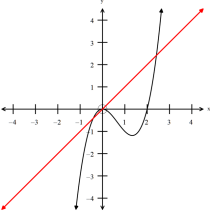
*x x*

− − − =

1 1 1 0

*x* =1 *or x* = 2

✔

(b) (i) *x* ≤ 0 ✔✔ *Answers will vary* (2) (ii) *y* = *x* ✔ 

✔

(2)

(iii) ( ) ✔ (1) 1 − =

*f* 32 4

**END OF SECTION ONE**

6

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**Section Two**

6. (6 marks)

32

12 − + 3 +1 ∫*t* ***i*** *t* ***j*** *dt*

(a) ( ) ( )

3

⎡⎛ ⎞ ⎤ = ⎢⎜ − ⎟ + + ⎥

2

*t*

( )

3

22

*t t t*

***i j***

✔

⎣⎝ ⎠ ⎦

1

⎛⎛ ⎞ ⎞ ⎛⎛ ⎞ ⎞ = ⎜⎜ − ⎟ + ~~+~~ ⎟ − ⎜⎜ − ⎟ + + ⎟

9 1

( ~~)~~ ( )

6 ~~27 3 2~~ 1 1 ***i ~~j~~ i j***

✔

2 2

⎝⎝ ⎠ ⎠ ⎝⎝ ⎠ ⎠

= +

0 28

***i j***

✔

(3)

π + − ∫*sin t* ***i*** *cos t* ***j*** *dt*

(b) ( ( )) ( ( ))

2

03 3

π ⎡ ⎤ = ~~−~~ ⎢ + ⎥

( ) ( )

*cos t sin t*

3 3

2

✔

**i j**

3 3

⎣ ⎦

0

⎛⎛ ⎛ π ⎞ ⎛ π ⎞ ⎞ ⎞ = ~~−~~ ⎜⎜ ⎜ ~~⎟~~ + ⎜ ⎟ ⎟ − + ⎟

1 3 3

( ( ) ( ) )

*cos ~~sin~~ cos sin* **i j i j**

0 0

✔

3 2 2

⎝⎝ ⎝ ⎠ ⎝ ⎠ ⎠ ⎠ 1

( )

= ~~−~~ − −

**j i**

3

1 1

= ~~+~~

**i j**

✔

3 3

(3)

7. (27 marks)

(a) (i) ***r***(*t*) = (10*cos*(*t*))***i*** +(10 *sin*(*t*)) ***j*** ( ) ( )

✔

*x cos t y sin t*

= =

10 10

( ) ( )

2 2

*sin t cos t*

+ =

1

2 2

*x y*

⎛ ⎞ ⎛ ⎞ ∴ ~~+~~ = ⎜ ~~⎟ ⎜~~ ⎟

1

10 10

⎝ ⎠ ⎝ ⎠

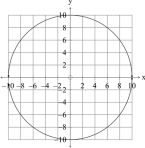
2 2

*x y*

+ =

100

✔

✔

(3)

7

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(ii) ***r***(*t*) = (10*cos*(*t*))***i*** +(10 *sin*(*t*)) ***j*** ( ) ( ( )) ( ( )) ***v i j***

*t sin t cos t*

= − +

10 10

✔

( )

( )

⎛ ⎞ ⎛− ⎞ ∙ = ⎜ ⎟ ∙ ⎜ ⎟

10 10

*cos t sin t*

( ) ( )

***r v***

*t t*

( )

( )

10 10

*sin t cos t*

⎝ ⎠ ⎝ ⎠

✔

( ) ( ) ( ) ( ) ( ) ( ) ***r v***

*t t cos t sin t sin t cos t* ∙ = − + =

100 100 0

π≠ ≠ ~~∴= ⇒=~~✔

( ) ( ~~) ( )~~ ***r , v***

*t t cos ~~t t~~*

0 0 ~~0~~ 2

Therefore the position vector is always at right angles to the velocity

vector.

(3)

( ) ( ) ( ( )) ( ( )) iii *t cos t sin t* ***a i j***

= − + −

10 10

✔

( ) (( ( )) ( ( )) ) ***a i j***

*t cos t sin t*

= − +

10 10

✔

( ) ( )

***a r***

*t t*

= −

***r***(*t*) is a position vector, i.e. it goes out from the origin.

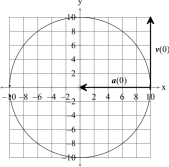
Therefore is directed towards the origin. (3) ***a*** (*t*) ✔

(iv) ( ) ( ) ( ) 10 0 10 , , ⎛ ⎞ ⎛ ⎞ ⎛− ⎞ = ⎜ ⎟ = ⎜ ⎟ = ⎜ ⎟ ***r v a***

0 0 0

0 10 0 ⎝ ⎠ ⎝ ⎠ ⎝ ⎠

✔

✔

✔

✔

(4)

(v) Speed = ***v*** (*t*)

✔

2 2

( ) ( ( )) ( ( ))

✔

***v***

*t sin t cos t* = − + 10 10

( ( ) ( ))

2 2

= + 100

*sin t cos t*

=

10 1

=

10

✔

The speed is constant. (3)

8

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(b) (i) ( ) ( ( )) ( ( )) 3 3 ***r*** *t* = *sin t* ***i*** + *cos t* ***j***.

( ) ( ( )) ( ( ))

3 3 0 0 0

***r i j***

= +

*sin cos*

0 0

⎛ ⎞ ⎛ ⎞ = ⎜ ⎟ = ⎜ ⎟

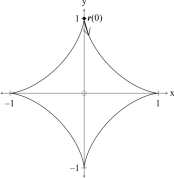
+

( ) ( )

+

***r r***

0 0

1 1 

−

⎝ ⎠ ⎝ ⎠

✔ ✔ ✔

(3)

(ii) ( ) ( ( ) ( )) ( ( ) ( ))✔✔✔ (2) 2 2 ***v*** *t* = 3*sin t cos t* ***i*** − 3*cos t sin t* ***j*** -1/error

1

⎛ ⎞

⎜ ⎟

***. r.***

2 2 0 35

⎛ π ⎞ ⎛ ⎞

✔

⎜ ⎟ = ⎜ ⎟ ≈ ⎜ ⎟

(iii)

4 1 0 35

⎝ ⎠ ⎜ ⎟ ⎝ ⎠

⎜ ⎟

2 2

⎝ ⎠

***. v.***

1 06

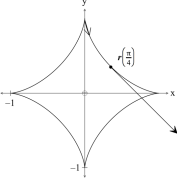
⎛ π ⎞ ⎛ ⎞

✔

⎜ ⎟ ≈ ⎜ ⎟

⎝ ⎠ ⎝− ⎠

4 1 06

✔

(3)

9

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(iv) t = ? ( ) 00 =⎛ ⎞

***v*** *t*

⎜ ⎟

⎝ ⎠

( ) ( ( ) ( )) ( ( ) ( ))

2 2

***v i j***

*t sin t cos t cos t sin t* = −

3 3

( ( ) ( )) ( ) ( )

2

*x sin t cos t sin t sin t* = =

3 1 5 2

***.***

( ) ( )

*If x sin t or sin t*

= = =

0 2 0 0

2 0 2 0 2

*t t*

= π π = π π

, , , , ...

π

✔

*If ~~x t~~*

= ~~=~~ π

0 ~~0~~

, ,

2

( ( ) ( )) ( ) ( )

2

*y cos t sin t sin t cos t*

= =

3 1 5 2

***.***

( ) ( )

*If y sin t or cos t*

= = =

0 2 0 0

π π

3

2 ~~0 2~~

*t ~~t~~*

= ~~π π =~~

, ~~, ,~~ ...

2 2

π

✔

*If ~~y t~~*

= ~~=~~ π

0 ~~0~~

, ,

2

0

⎛ ⎞

r *t* , first ~~time is~~ *t*π

( ~~)~~

***v***

*t*

=

> ~~=~~

02

So for fo

⎜ ~~⎟~~

✔

0

⎝ ⎠

(3)

8. (3 marks)

**AG AO OG OA OG** = + = − +

✔

(a) (2) = − +

**a g**

✔

(b) (1) ( ~~)~~ ( ) 1 1 1 **OM** = **OA** + **AG** = **a** + ~~−~~**a** + **g** = **a** + **g**

✔

2 2 2

9. (13 marks)

✔

2 2 2 2 *C* −1***,***4***,***0 *r* = 1− −1 + 2 − 4 + 4 − 0 =4+4+16=24(a) (i) ( ) ( ( )) ( ) ( ) ✔

( ) ( ) (3) 2 2 2

✔

*x* +1 + *y* − 4 + *z* = 24

⎛− ⎞ ⎛ ⎞

4 0

⎜ ⎟ ⎜ ⎟ = = −

**PQ** *,* **PR**

4 1

(ii)

✔

⎜ ⎟ ⎜ ⎟

⎝ − ⎠ ⎝− ⎠ 8 3

⎛ ⎞ ⎛− ⎞ ⎛ ⎞

1 4 0

⎜ ⎟ ⎜ ⎟ ⎜ ⎟ = + + − **r** *t s*

1 4 1

✔✔

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎝ ⎠ ⎝ − ⎠ ⎝− ⎠ 1 8 3

*Other solutions are possible* (3)

10

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⎛− ⎞ ⎛ ⎞

4 0

⎜ ⎟ ⎜ ⎟ = = −

**PQ** *,* **PR**

4 1

(iii)

⎜ ⎟ ⎜ ⎟

⎝ − ⎠ ⎝− ⎠ 8 3

⎛− ⎞

20

⎜ ⎟ × = −

**PQ PR** 12

(1)

✔

4

⎝ ⎠

4 2 5

*.*

⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ = + −

(b) (i) ( )

**r***bird*

*t t*

5 1

⎜ ⎟ ⎜ ⎟

⎝ ⎠ ⎝ − ⎠

6 3

*x . t, y t, z t*

= + = − = −

4 2 5 5 6 3 *If z , t*

= =

0 2

check *x =*

4 5 9

+ =

*z*

= − =

6 6 0

So at (9,0,3) *t* = 2 ✔ (1)

(ii) to Mouse takes 1 second to get to its hole. (9***,***3***,***0) (9***,***4***,***0) ✔ (1) 2 5 2 5

***. .***

⎛ ⎞ ⎛ ⎞

✔

⎜ ⎟ ⎜ ⎟ − = = (iii) ✔

1 4 03 0 3 91 ***. .***

m/s m/s

⎜ ⎟ ⎜ ⎟

− − ⎝ ⎠ ⎝ ⎠

3 3

Change in speed is 0.12 m/s ✔ (2)

(iv) At the bird is at *t* =1 *P*(6**.**5**,** 4**,** 3) 6 5 2 5

*. .*

⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ = +

( )

**r***bird*

*t t*

4 0

⎜ ⎟ ⎜ ⎟

⎝ ⎠ ⎝ − ⎠

3 3

After one second (when the mouse gets to its hole) ✔

⎛ *.* ⎞ ⎛ *.* ⎞ ⎛ ⎞

6 5 2 5 9 ⎜ ⎟ ⎜ ⎟ ⎜ ⎟ = + =

( ) the bird arrives at the nest, **r***bird*

1 4 1 0 4 ⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎝ ⎠ ⎝ − ⎠ ⎝ ⎠ 3 3 0

✔

so they both arrive at the hole together.

Let’s hope the mouse does not have a long tail!!! (2)



11

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10. (12 marks)

(a)

⎛ ⎞

6

⎛ ⎛ π ⎞⎞ ⎛ ⎛ π ⎞⎞ ⎛ π ⎞ 6

1 2

*i ~~cis~~ cis ~~cis~~*

( ~~)~~

+ ⎜ ⎟ ⎜ ⎜ ~~⎟~~⎟ ⎜ ⎜ ~~⎟~~⎟ ⎜ ⎟ 2 4 2

⎝ ⎠ ⎜⎝ ⎝ ⎠⎠ ⎝ ⎠⎟

⎜ ⎟ =

*Re ~~Re~~*

2 2

( ~~)~~

124

− ⎛ ⎛ π ⎞⎞ *icis*

⎜ ⎟ ⎜ − ⎟ ⎜ ⎜ ⎜ ⎟⎟ ⎟ ⎝ ⎠ ⎝ ⎝ ⎝ ⎠⎠ ⎠

⎛ ⎛ π π π ⎞⎞ = ⎜ ⎜ + ~~+~~ ⎟⎟

8 6 2

*Re ~~cis~~*

✔

2 4 2 4

⎝ ⎝ ⎠⎠

⎛ ⎛ π ⎞⎞ = ⎜ ⎜ ⎟⎟

5

42

*Re ~~cis~~*

⎝ ⎝ ⎠⎠

⎛ ⎛ π ⎞⎞ = ⎜ ⎜ ⎟⎟

42

*Re ~~cis~~*

✔

⎝ ⎝ ⎠⎠

⎛ π ⎞ ⎛ π ⎞ = ⎜ ~~⎟~~ + ⎜

⎛ ⎞

42 2

*Re ~~cos isin~~*

⎜ ⎟⎟

⎝ ⎠ ⎝ ⎠

⎝ ⎠

4 (0 )

= +

*Re i*

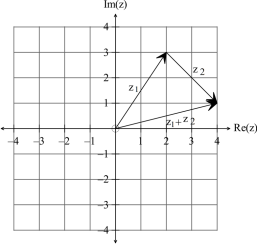
✔

=

0

(3)

(b)

✔ 

✔

(2)

12

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(c) (i) 2 3 1 5

. + +

*i i*

*x ~~yi~~*

+ ~~= −~~

1 3

+ −

*i i*

2 3 1 5 2 3 1 1 5 3

+ + + − + +

*i i i i i i*

✔ ✔

− ~~= × − ×~~

1 3 1 1 3 3

+ − + − − +

*i i i i i i*

2 2

2 3 2 3 3 15 5

+ − − + + +

*i i i i i i*

✔ ✔

= ~~−~~

2 2

1 9

− −

*i i*

+ ⎛ − + ⎞ = ~~−~~ ⎜ ⎟

5 2 16

*i i*

2 10

⎝ ⎠

5 1 1 8

⎛ ⎞ = ~~+ + −~~ ⎜ ⎟

*i*

2 5 2 5

⎝ ⎠

27 11

*i*

= ~~−~~

10 10

*x and y*

= = −

2***.***7 1***.***1

✔✔

(6)

(ii) *x* + *yi* = 4 + 3*i x* = 2*.*12*, y* = 0*.*71 ✔ (1)

11. (6 marks)

2 3

⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ − ∙ −

3 2

⎜ ⎟ ⎜ ⎟

1 4

⎝ ⎠ ⎝ ⎠ θ ~~=~~

(a) ( ~~)~~ ✔

*cos*

2 3

⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ − −

3 2

⎜ ⎟ ⎜ ⎟

1 4

⎝ ⎠ ⎝ ⎠

( ~~)~~ 6 6 4

+ +

*cos*

θ ~~=~~

4 9 1 9 4 16

+ + + +

16

=

14 29

✔

o

θ =

37 43

*.*

(2)

(b) The projection of (2) 1629

***a• b***

***a ~~b~~***

on = ~~=~~

***b***

✔ ✔

4

3 4

⎛ ⎞

⎛ ⎞ ⎛ ⎞

⎜ ⎟ =

⎜ ⎟ ⎜ ⎟ − ∙ =

(c) ✔✔ *Answers will vary* (2) **p**

0

2 0 0

⎜ ⎟ ⎜ ⎟

⎝− ⎠

3

⎝ ⎠ ⎝− ⎠

4 3

13

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12. (17 marks)

(a) ( ( )) ( )( ) ( ) ( )

2

*p q x* = *x* +1 *x* + 3 and *p x* = *x* −1 find *y* = *q x .*

2

✔

( ( )) ( ( ))

*p q x q x*

= −

1

( ( )) ( )( )

*p q x x x*

= + +

1 3

2

= + +

*x x*

4 3

✔

2

= + + −

*x x*

4 4 1

2

✔

( ( )) ( )

*p q x x*

= + −

2 1

∴ *q*( *x*) = *x* + 2

✔

(4)

(b) (i) *f* ( *x*) = *x* ( *x* −1)( *x* +1) ✔ (1) (ii) *f* ( *x*) = *x* ( *x* −1)( *x* +1) ✔✔ (2)

1

(iii) ( ~~)~~ ✔✔✔ (3) ( )( )

*f ~~x~~*

*x x x* =− +

1 1

(c) (i) 2 *f* (1) = 2×(−3) = −6 ✔ (1) (ii) *f* ( −1 ) = *f* (1) = −3 ✔ (1)

(iii) ( ) ✔ (1) 1

− = − =

*f* 3 1 1

(iv) True ✔

(as *f* (−1) = 3 and *f* (1) = −3

and *f -1* is monotonically decreasing) (1)

(v) True ✔ (1)

(d) (i) ( ) 2*x* 2*x f x* = *e* ⇒*y* = *e*

*y x e*

2

=

To get inverse

( )

2

*y ln x*

=

( )

*ln x*

( ~~)~~

−

1

*y ~~f x~~*

= ~~=~~

2

✔ (1)

(ii) ( ( ( ~~)~~)) ( ~~)~~ ✔ (1) ( ) 1 1 1 1

*ln*

*f f f ~~f~~* − − − = ~~=~~ =

1 ~~1~~ 0

2

14

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13. (4 marks)

(a) ( ( )) ( ) ( ( ))2 2

*f g x* = *f x* = 1− *x* −1≤ *x* ≤1 0 ≤ *f g x* ≤1

✔ ✔ (2)

(b) (i) ( ) =1+*x h x e*

To get inverse:

*y*

*x e*

= +

1

*y*

*x e*

− =

1

( )

*ln x y*

− =

1

( ) ( )

−

1

*y h x ln x*

= = −

1

✔ (1) (ii) ( ) ( )

1 *h* 2 *ln* 2 1 0

− = − =

✔ (1)

14. (4 marks)

⎧− + ≥ = ~~− + =~~ ⎨⎩ + <

2 2 0

*x*

for

*x*

(a) (1)

*y ~~x~~*

2 ~~2~~

2 2 0

*x*

for

*x*

⎧ − ≤

1 1

*x x*

for

(b) ✔ (1)

*y ~~x~~*

= ~~− =~~ ⎨⎩ − >

11 1

*x x*

for

(c)

*For x x x For x x x*> − + = − < < −+=−

1 2 2 1 0 1 2 213 3 1

= =

*x x*

1

=

*x*

✔

*For x x x*

< + = −

0 2 2 1

3 1

*x*

= −

1

✔

*x*

= ~~−~~

3

(2)

15. (3 marks)

*a* = −1*, b* = −2*, c* = 0

✔ ✔ ✔ (2)

15

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16. (5 marks)

Prove that ( ) ( ) ( ) ( )

5 3 *cos* 5θ =16*cos* θ − 20*cos* θ + 5*cos* θ ( ) ( ( ))

*cos Re cis*

5 5

θ = θ

5

( ( ) ( ))

= θ + θ

*Re cos isin*

✔

2

( ( ) ( )( ( )) ( )( ( ))

5 4 3

= θ + θ θ + θ θ*Re cos cos isin cos isin* 5 10

3 4 5

( )( ( )) ( )( ( )) (()))

2

+ θ θ + θ θ +θ10 5

*cos isin cos isin isin*✔

( ) ( ) ( ) ( ) ( )

5 3 2 4

= θ − θ θ + θ θ *cos cos sin cos sin* 10 5

✔

( ) ( )

2 2

*BUT sin c os*

θ = − θ

1

2

( ) ( ) ( ) ( ) ( ) ( )θ = θ − θ ⎡ − θ ⎤ + θ ⎡ −θ⎤

5 3 2 2

*cos cos cos c os cos c os*

5 10 1 5 1 ⎣ ⎦ ⎣ ⎦

✔

= θ − ( ) ( ) ( ) ( )()( )

θ + θ + θ ⎡ − θ+θ⎤⎣ ⎦

5

3 5 2 4

*cos cos*

10

10 5 1 2

*cos cos c os cos*

( ) ( ) ( ) ( ) ( )

5 3 3 5

= θ − θ + θ − θ +θ11 10 5 10 5

*cos cos cos cos cos* ✔

( ) ( ) ( ) ( )

5 3

*cos cos cos cos*

5 16 20 5

θ = θ − θ + θ (5)

**END OF SECTION TWO**

16